1: Translation into propositional logic (10 points) Translate the following sentences into *propositional logic*. Atomic sentences are represented by uppercase letters. Do not forget to provide the translation key — one key for the whole exercise. Represent as much logical structure as possible.

a. We will not bake a cake unless the oven is already clean and there is flour at home.

b. Neither is the oven already clean nor is there oven cleaner at home.

2: Translation into first-order logic (10 points) Translate the following sentences to *first-order logic*. Do not forget to provide the translation key — one key for the whole exercise. Represent as much logical structure as possible and let the domain of discourse be the set of all people.

a. If Donald complains about Joe, then he is either not letting Joe talk or he ignores Joe, and he lies.

b. Donald complains about Joe, but Joe ignores him.

3: Formal proofs (30 points) Give formal proofs of the following inferences. Do not forget to provide justifications in the correct order. You may only use the Introduction and Elimination rules and the Reiteration rule.

a.

$(\neg A \to A) \leftrightarrow (\neg A \to \bot)$

b.

$P \to \neg R$ $P \vee Q$
$\neg Q \to \neg R$

c.

$a \neq b$ $b = c$ $c = d$
$a \neq d$

4: Truth tables (15 points) Use *truth tables* to answer the next questions. Provide the full truth tables. Order the rows in the truth tables as follows:

P	Q	R	...
T	T	T	...
T	T	F	...
T	F	T	...
T	F	F	...
F	T	T	...
F	T	F	...
F	F	T	...
F	F	F	...

a = c	b = a	b = c	...
T	T	T	...
T	T	F	...
T	F	T	...
T	F	F	...
F	T	T	...
F	T	F	...
F	F	T	...
F	F	F	...

a. Is $(\neg(P \land \neg Q) \rightarrow (Q \rightarrow \neg R)) \leftrightarrow (P \lor \neg R)$ a *tautology*?
Do not forget to draw an explicit conclusion from the truth table and to explain your answer.

b. Is the sentence $\neg(a = c) \lor (\neg(b = a) \rightarrow \neg(b = c))$ a *logical truth*?
Draw an explicit conclusion, explain your answer and indicate the spurious rows.

5: Normal forms of propositional logic (15 points)

a. Provide a negation normal form (NNF) of this sentence: $((P \vee \neg Q) \vee R) \leftrightarrow (\neg R \wedge Q)$. Indicate the intermediate steps. You do *not* have to provide justifications for the steps.

b. Provide a conjunctive normal form (CNF) of this sentence:

$$P \to \neg(\neg Q \vee \neg(R \wedge \neg S))$$

Indicate the intermediate steps. You do *not* have to provide justifications for the steps.

6: Set theory (10 points) Consider the following sets:

$$A = \{a, \{b, c\}\}, \;\; B = \{\emptyset, \{b, c\}\}, \;\; C = \{b, c\}, \;\; D = \{a\} \;\; \text{and} \;\; R = \{\langle a, b\rangle, \langle a, c\rangle\}.$$

Provide a list of five true and a list of five false statements in the language of set theory, clearly indicating which statements are true and which are false.

You should use each of the sets above at least twice and each of the following symbols at least once.

$$\cap \qquad \cup \qquad \setminus \qquad \subseteq \qquad \subsetneq \qquad \times \qquad \emptyset \qquad \in$$

7: Bonus question (10 points) Give a formal proof of the following inference:

$$\begin{array}{|l} A \lor B \\ \hline (A \land B) \lor (\neg A \leftrightarrow B) \end{array}$$

Do not forget to provide justifications in the correct order. You may only use the Introduction and Elimination rules and the Reiteration rule.